So if we denote by ϵ the difference between the difference quotient and the derivative, we obtain

$$\lim_{\Delta x \to 0} \varepsilon = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x} - f'(a) \right) = f'(a) - f'(a) = 0$$

But

$$\varepsilon = \frac{\Delta y}{\Delta x} - f'(a)$$
 \Rightarrow $\Delta y = f'(a) \Delta x + \varepsilon \Delta x$

Thus, for a differentiable function f, we can write

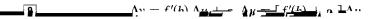
$$\boxed{7} \qquad \Delta y = f'(a) \ \Delta x + \varepsilon \ \Delta x \qquad \text{where } \varepsilon \to 0 \ \text{ as } \Delta x \to 0$$

This property of differentiable functions is what enables us to prove the Chain Rule.

Proof of the Chain Rule Suppose u = g(x) is differentiable at a and y = f(u) is differentiable at b = g(a). If Δx is an increment in x and Δu and Δy are the corresponding increments in u and y, then we can use Equation 7 to write

$$\Delta u = g'(a) \Delta x + \varepsilon_1 \Delta x = [g'(a) + \varepsilon_1] \Delta x$$

where $\varepsilon_1 \to 0$ as $\Delta x \to 0$. Similarly



$\int 12 v = \cos(a^3 + x^3)$	14 $v = a^3 + \cos^3 r$	51) Find all points on the graph of the function			
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59. Use the table to estimate the value of h'(0.5), where h(x) = f(g(x)).

x	0	0.1	0.2	0.3	0.4	0.5	0.6
f(r)	_126	1/1 8	18.4	23.0	25.0	27.5	20.1

modeled by the product of an exponential function and a sine or cosine function. Suppose the equation of motion of a point on such a spring is

$$s(t) = 2e^{-1.5t}\sin 2\pi t$$